Assignment 3 : Design of a Measurement System

ES21Q Design of Measurement Systems

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25/04/02

Abstract

A torque sensor was designed to measure the torque applied to a test rig consisting of a hollow steel bar connected to a frame. The system was based on strain gauges and was designed to give an automatic reading of the torsion applied to the bar. Equations were produced to model the behaviour and strains produced in the bar and 4 strain gauges were mounted on the bar. The gauges were connected in a whetstone bridge configuration and the output from this circuit was derived. An operational amplifier was used to amplify the bridge output voltage and various display and calibration options were discussed. The system was not assembled but the design was brought to a stage where prototyping could take place. Limitations and improvements to the design were discussed.

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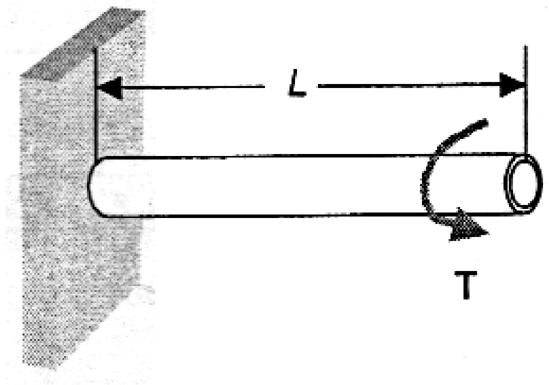
Introduction/Specification

A torque sensor is required to measure the torque applied to a test rig consisting of a hollow steel bar connected to a frame as shown below:



[1]

The diameters are: Length = 200mm, Outside diameter d_o = 20mm. Internal diameter d_i = 17mm. Torque can be applied via a thin steel bar attached to the free end and weight applied at the other end. The test rig is to be used to assemble a system to give an automatic measurement of torsion. The sensor will be based on strain gauges mounted on the bar. These gauges will measure the strain in the bar due to the torsion produced by the torque applied. The final system design should calibrated be able to give an automatic measurement of the torque applied. Torque measuring devices have many applications, mostly in industry, such as determining power of motors and testing tightness of screws/bolts in automatic assembly systems.

Design

Gauge Selection

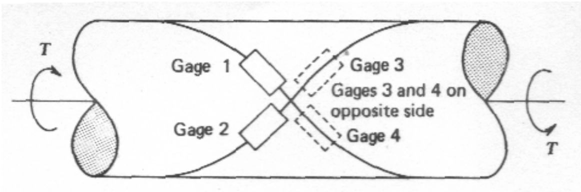
Many factors can influence the type of strain gauge used and choosing a type of strain gauge is probably the primary concerns when designing a strain measurement system. Generally choosing a strain gauge is a compromise between many factors. Cost is not usually a primary factor since the unit cost is low compared to the installation cost. Some of the more important factors in strain gauge selection include: Gauge series (Material used), pattern, length, fatigue life and resistance. In this case we have few special requirements and the primary choice is length. The gauges specified in the assignment sheet (see appendix A) come in 2mm and 5mm sizes. The bar they will be mounted on is 20 mm in diameter. Advantages of using a longer gauge include greater grid area for better heat dissipation and easier handling and installation. They also have improved strain averaging but this mainly applies to inhomogeneous materials such as fibre-reinforced composites. The 5mm gauge in the assignment sheet will be chosen since there is enough room to accommodate it and it will allow for more accurate installation which will reduce error due to misalignment. This has a gauge factor of 2.00 and a resistance of 120Ω±0.5%. The principle reasons for choosing the smaller gauge, restricted room and localised strain do not apply in this case. The

[1] Source: Assignment Sheet

material copper nickel alloy is suitable for this application and will retain its reference point.

Mounting of Gauges

Torque is measured by either sensing the actual shaft deflection caused by a twisting force, or by detecting the effects of this deflection. In this case strain gauges will be used. mounted in pairs on the shaft, one gauge measuring the increase in length (in the direction in which the surface is under tension), the other measuring the decrease in length in the other direction. The gauge will therefore be oriented so that their gridlines are at 45° to the shaft axis and the gauges are at an angle of 90º apart. To minimise effects of temperature, forces and moments other than those being measures and to give the maximum output voltage 4 strain gauges will be used two measuring the quantities as described and another pair measuring the same quantities on the opposite side of the bar. The following diagram shows the mounting of the strain gauges on the bar:



Positioning of strain gauges for torque measurement[2].

The accurate orientation of the gauges is vital to reduce error. They will also be placed in the centre of the bar. They will be attached to the steel surface with epoxy resin.
The gauges will be wired in a whetstone bridge configuration to powered by a 9V battery. A whetstone bridge is used to convert a change in resistance to an output voltage and produce maximum accuracy. The configuration is shown below:
The output of this bridge circuit is given by:

$$\Delta V_o = \left(\frac{R_1 + \Delta R_1}{R_1 + \Delta R_1 + R_2 + \Delta R_2} - \frac{R_4 + \Delta R_4}{R_3 + \Delta R_3 + R_4 + \Delta R_4} \right) V_s$$

Strain Gauge R=1k

However in this system R_1, R_2, R_3, and R_4 are all strain gauges and $R_1 = R_2 = R_3 = R_4$. Also the magnitude of ΔR will be the same for all the strain gauges since the effect is either the same or opposite (compression of tension depending of applied torque and orientation of strain gauges. Therefore:

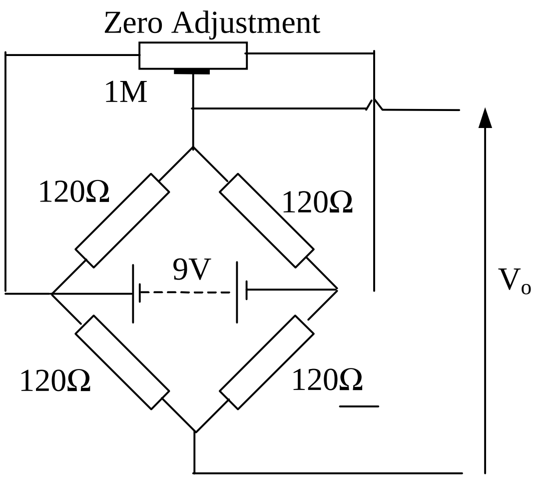
$$\Delta V_o = \left(\frac{R + \Delta R}{R + \Delta R + R - \Delta R} - \frac{R - \Delta R}{R + \Delta R + R - \Delta R} \right) V_s$$

$$\Delta V_o = \left(\frac{\Delta \mathrm{R}}{\mathrm{R}} \right) V_s$$

Note that this is twice as sensitive as a single active gauge device but half as sensitive as a 4 active device. A linear relationship should be observed between ΔV_o and ΔR.
The bridge will be balanced if $R_1R_3 = R_2R_4$. In theory, with the given circuit the circuit should be balanced without the need for the potentiometer. However factors such as component tolerances, strain gauge mounting and temperature differentials will mean the circuit will not be balanced so the potentiometer is included, as shown, to set the output voltage to zero. Since one end of the bar is fixed the wires from the strain

[2] *Source: Dally, James W., Instrumentation for engineering measurements, Wiley, 1993.*

gauges can be taken straight to the processing electronics without the need for slip rings as required in many practical torque sensors applications.



Bridge Circuit

Relationship between Shear Stress and Torque

The strain produced at the surface of the bar will be caused by the shear stress produced by the torque applied to the end of the bar. The shear stress produced is required as a function of the applied torque.
A torque applied to the free end of a solid bar can be considered to rotate the end of the bar by a small angle θ. The bar will distort accordingly.
Taking a reference point on each end of the bar the angle from the reference point at the fixed end to the displaced point at the free end after a torque has been applied will be the shear strain. The shear modulus G is defined as (shear stress, τ)/(shear strain, γ):

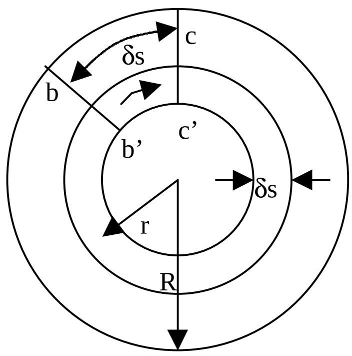
$$G = \frac{\tau}{\gamma}$$

The arc length that the reference point will move through when a torque is applied is given by $l_{arc} = R\theta$. Or by considering the surface geometry, $l_{arc} = L\gamma$, where L is the length of the bar. Combining these two equations gives:

$$\gamma = \frac{R\theta}{L}$$

However we need the shear stress in terms of applied torque. Consider a small segment of cross-section (bcb'c'), thickens δr and side length δs. Assume the centre of the segment is at a radius r from the longitudinal axis:

Consider the stress along the face of segment to be τ. The total force along the face is given by:
Force = $\tau \times \delta r \times \delta s$
And hence the moment is given by:
Moment = $(\tau \times \delta r \times \delta s) \times r$.



Cross Section

The torque exerted by all the segments at radius r from the centre is:

Torque exerted $= \tau \times \delta r \times r \times \sum \delta s$

$= \tau \times \delta r \times r \times 2\pi r$

$= \tau 2\pi r^2 \delta r$

The expression for the total torque (summation of all ring torques) is given below:

$$T = \int_0^R \tau 2\pi r^2 dr$$

Combine with earlier equation, $G = \dfrac{\tau}{\gamma}$, giving:

$$T = \frac{G\theta}{L}\int_0^R 2\pi r^3 dr$$

But:

$$J = \int_0^R 2\pi r^3 dr$$

(polar second moment of area)

So: $\dfrac{T}{J} = \dfrac{G\theta}{L} = \dfrac{\tau}{r}$ and $\tau = \dfrac{rT}{J}$

We however are considering a hollow bar so the limits for the integral are different:

$$J = 2\pi \int_{r_i}^{r_o} r^3 dr = \frac{\pi(r_o^{\,4} - r_i^{\,4})}{2}$$

Giving a final expression for the shear stress in our system as:

$$\tau = \frac{2r_o T}{\pi(r_o^{\,4} - r_i^{\,4})}$$

τ: Shear strain.
T: Applied Torque
r_o: Outer radius.
r_i: Inner radius.

The relationship between the shear strain and the shear stress is $\gamma = \dfrac{\tau}{G}$ where $G = \dfrac{E}{(1+v)}$

v: Poissons ratio
E: Young's Modulus

As shear stresses are equivalent to the normal stresses then the normal strain is:

$$\varepsilon = \tau \times \left(\frac{1+v}{E}\right).$$

So the strain of our system expressed as a function of torque is:

$$\varepsilon = \frac{2r_o T(1+v)}{\pi E(r_o^{\,4} - r_i^{\,4})}$$

From the properties of strain gauges:

$$\frac{\Delta R}{R} = G\varepsilon$$

So the change in resistance of a single strain gauge is given by:

$$\Delta R = \frac{2GRr_o T(1+v)}{\pi E(r_o^{\,4} - r_i^{\,4})}$$

This could be positive or negative depending on placement. From previous diagram:

$$\frac{\Delta R_1}{R_1} = -\frac{\Delta R_2}{R_2} = \frac{\Delta R_3}{R_3} = -\frac{\Delta R_4}{R_4} = \frac{2Gr_oT(1+v)}{\pi E(r_o^{\,4} - r_i^{\,4})}$$

From previous calculation the output from the Whetstone Bridge is:

$$\Delta V_o = \left(\frac{\Delta \mathrm{R}}{\mathrm{R}}\right)V_s$$

So the output of the sensor is:

$$\Delta V_o = \frac{2r_oT(1+v)}{\pi E(r_o^{\,4} - r_i^{\,4})}GV_s$$

ΔV_o: Change in output voltage.
V_s: Bridge input voltage
T: Applied Torque
r_o: Outer radius.
r_i: Inner radius.
v: Poissons ratio
E: Young's Modulus
G: Gauge factor

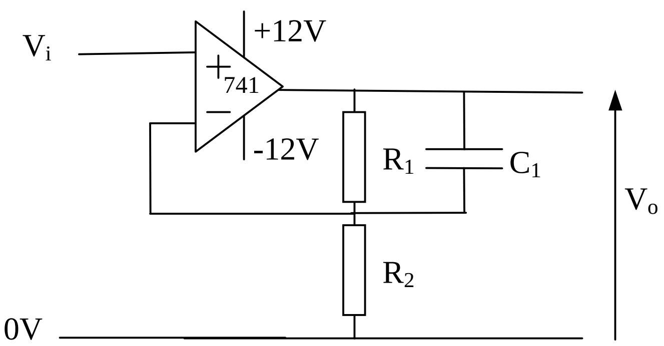
Using some sample figures an approximate output voltage level can be found. For a torque of T = 1Nm and:
V_s = 9V, d_o = 20mm, d_i = 17mm, E_{steel} = 210GPa (data book) G = 2.00 (data sheet). v = 0.292 (data book).

$$\Delta V_o = \frac{2\times 0.010\times 1\times(1+0.292)}{\pi\times 210\times 10^9\times((0.010)^4 - (0.0035)^4)}\times 2.00\times 9 = 71.5\text{mV}$$

A voltage of 71mV is a lot easier to measure and display. Also a considerably larger current can be drawn from the op-amp.

Amplifier Design

It can be seen for the test calculation that the output is in the order of tens of mV per Nm torque applied. This is not a suitable output direct display and so an amplifier is required to amplify this voltage for easy reading. The amplifier will be based on the 741 operational amplifier. A non-inverting amplifier will be used but this is not exclusive since reversal of the strain gauges (or output from the Whetstone Bridge) will cause the voltage to change polarity. Similarly applying torque in one direction will give a positive torque but if it is applied in the other direction a negative voltage will be produced. The op-amp will be powered by a ±15V supply. The circuit for a non-inverting amplifier based on the 741 is shown below:



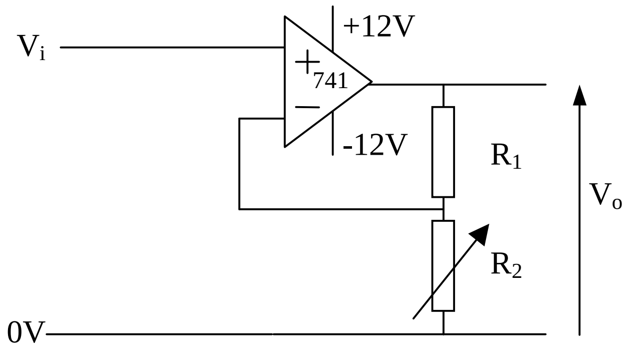
Non Inverting Amplifier:

$$G = \frac{R_1 + R_2}{R_2}$$

The gain of the amplifier should be about 1000 to give a voltage in a suitable range. This gain will give an output voltage of about 70mV/Nm. Choosing R_1 = 1M and R_2 = 1k gives a gain of 1001 which is ideal for this application. It may be necessary to use a low-pass filter to minimise noise. This can be done by connecting a capacitor across the feedback resistor, acting as a short circuit for high frequencies (found in noise). 100pF is probably a suitable value. This could be even higher is response time is not important.

Calibration and Data Presentation

The systems zero point can easily be set using the offset potentiometer connected to the whetstone bridge circuit. The potentiometer would be adjusted until there is no



output voltage. Calibration of the scale could be achieved by using a variable resistor across R_2 to control the gain of the amplifier. It would however probably be better to add another amplifier stage with a controllable gain. Therefore a better range could be used e.g.
The gain of the amplifier should be lower to give a fine adjustment. After the zero point is set a known torque can be applied to the bar. A voltmeter can be used in conjunction with the scale control to set the required voltage per unit torque. Calibrations would involve applying a known torque using weights/bar etc. The output would then be set to the desired value.

The output of the device could be displayed in a number of ways. The simplest is using a voltmeter but the scale could be hard to read. A needle and dial gauge (magnetic) could be used to give a reading. This is simple to set up and calibrate. The scale could be calibrated as previously described so it read in Nm. A digital voltmeter is the next step up, giving a numerical readout of the torque applied, again calibrated as described. Both these methods confirm to the specification of a dynamic display of torque. A data capture device could also be used such as an A/D card plugged into a PC running suitable software.

Conclusion

The design of the measuring system, with the use of the whetstone bridge circuit and four strain gauges means that the system is relatively insensitive to uniform temperature changes axial forces and bending force. For instance if the temperature changes the resistance of all the strain gauges will change by the same amount and the balance condition of the bridge will remain constant. Similarly if a force is applied to the end of the rod causing bending to take place the positioning and wiring of the strain gauges means that the corresponding strain gauge will be changed inversely, therefore compensating for the bending. However the gauges only have a thermally induced output of ±2 micron strain/°C for a temperature range of 20 to 160°C, and from ±5 micron strain/°C for a temperature range from 160 to 180°C. This should make little difference to our results even without temperature compensation. This is because the unit is not expected to undergo large temperature changes.

The design could be improved by improving the signal processing electronics to cope with non-linearities etc. High-quality transducers utilising strain gages as the primary sensing element incorporate sophisticated processing techniques to minimise thermal effects, nonlinearities, hysteresis, and other sources of error.

The design produced seems to conform to the specification given. A practical evaluation is now required to confirm the device functions as predicted.

Bibliography

Lecture Notes - ES21Q Design of Measurement Systems.
Assignment Sheet - ES21Q Design of Measurement Systems.
http://www.omega.com/literature/transactions/volume3/force3.html
Dally, James W., *Instrumentation for engineering measurements*, Wiley, 1993.
Hulse, R & Cain, J., *Structural Mechanics*, Palgrave, 2000.
University of Warwick Engineering Data Book.

Appendix A: Assignment Sheet

- Includes Strain Gauge Data Sheet.