Assignment 1 : Metrology

ES21Q Design of Measurement Systems

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Abstract

The aim of this laboratory was to improve understanding of measurement techniques and errors by measuring set dimensions of a given workpiece. The workpiece was measured using hand-tools, (micrometers, venire callipers, Height gauge, sine bar, block gauges and surface plates), and a CMM (co-ordinate measuring machine). The accuracy and uses of each material was considered and the most suitable tool was selected for each measurement. The dimensions were calculated form the results and analysed with respect to errors. The measurements using hand-tools and the CMM were compared and commented on. A large error of 0.1mm was noticed using the CMM and possible causes of this were discussed.

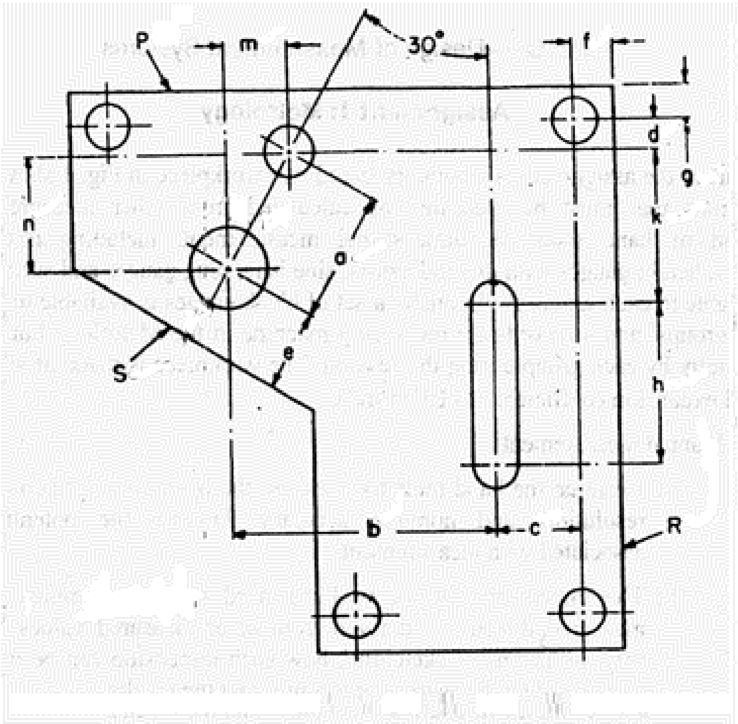
A CMM was recognised as the most accurate option but the high price and high setup times means hand tools are suitable for simple measurements that do not require the high accuracy of a CMM.

Introduction

The aim of this laboratory is to compare different types of measuring tool including CMM and a selection of hand-tools. A workpiece will be measures and the results compared. It will also be used to increase understanding of errors associated with types of measurement and the sources of these errors.

Part 1 – Measurement of Workpiece

The workpice to be measured consists of a steel block (thermal expansion co-efficient 11×10^{-6}mm/ºC with some holes and a slot drilled through it.



The diagram shows the shape of the workpiece and the dimensions to be measured. The measurement will be performed using hand tools. The tools available are, micrometers, venire callipers, Height gauge, sine bar, block gauges and surface plates. Plug gauges are provided with the workpiece. The measurements will then be made with a CMM (co-ordinate measuring machine), both sets of measurements will then be compared.

1/ Manual Measurement

First the accuracy of each of the tools available must be found. The workpiece must be measured as accurately as possible, so the right (and most accurate) tool must be selected for each measurement.

Tool	Range	Accuracy
Micrometer	0-25mm 25-50mm 0-25mm (depth)	±0.01mm ±0.01mm ±0.01mm
Vernier Callipers	0-150mm	±0.05mm

Height Gauge	40-360mm *	±0.02mm
Slip Gauges	0-100mm	±0.01μm

As can be seen from the table the slip gages are the most accurate available. However these cannot be used directly for most of the measurements required. The next most accurate is the micrometers, followed by the Height Gauge. Improved accuracy is achieved on the height gauge since a pressure sensor is used that allows the same pressure to be applied each time, thereby reducing errors due to elastic deformation.

The micrometer measures dimensions using a screw gauge and two contacts. Constant pressure is applied over multiple measurements by the use of a special device on the end of the screw.
The callipers measure a workpiece by using a sliding scale, a dial and two pincers. It has two sets of pincers for internal and external measurements.
The height gauge measures a dimension by using a flat base and a moveable scale with an arm and pressure sensor. The fine adjustment is made using a screw and the pressure gauge is used to apply the same pressure each time.

All the tools will suffer from multiple types of error.

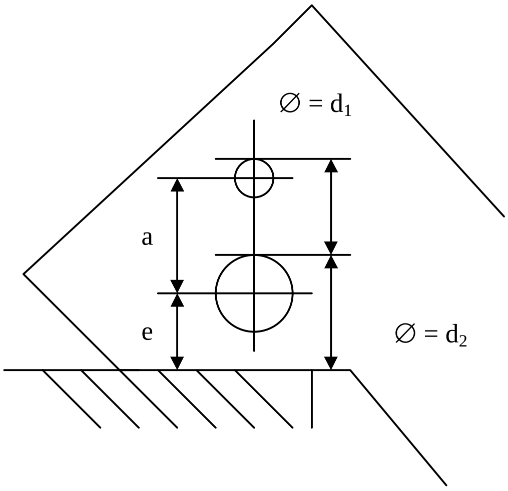
- Parallax errors are caused by the eye not lining up the marks on the scale accurately. This will apply to all tools but the slip gauges.
- Elastic deformation will be caused by pressure applied to the object by the measuring device.
- Systematic errors will occur due to improper calibration, or user error in setting up the zero/measuring points.
- Errors due to ambient conditions will apply to all the tools. The most noticeable of these is thermal expansion which results in the length of a material changing as the temperature is changed.
- Abbe’s offset errors will apply to all tools where the line of measurement is not collinear to the dimension being measured. This effect increases as the offset increases. This applies to all the measurement tools but the micrometers.
- Cosine errors will occur where the measurement instrument is misaligned relative to the workpiece.
- Plug gauges will contribute to the errors since they must be slightly smaller than the hole to fit in.

Letters refer to the dimensions shown on the diagram (when applicable).

Since the height gauge will be used to measure many of the dimensions the reading on the height gauge at the workbench. The pressure to be used should also be set as zero on the dial so the same pressure can be used for every measurement. The offset measured will apply to all measurements. This reference was measured as 46.40mm.

* Due to the arm and pressure sensor a relative measurement can be taken. Therefore it is possible to measure from 0mm.

The first measurements to be taken are 'a' and 'e'. The distance from the side to the centre of the first hole and form the centre of that hole to the centre of the second hole. To do this plug gauges are used in the holes. The plug gauges used are first measured using the micrometer.



d_1=10.01mm d_2=12.01mm (micrometer)

1st height reading is measured as 87.86mm

$\therefore$ $a+e+r_1$ = 87.86 - 46.40

2nd height reading is 67.36mm

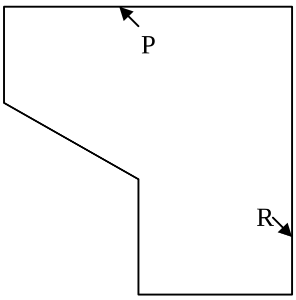
$\therefore$ $e+r_2$ = 67.36 - 46.40

e = 14.955mm
Estimated Accuracy: 2 Height Gauge 1 micrometer.
0.02+0.02+0.01 = ±0.05mm
e = 14.955 ± 0.4% mm

a = 87.86-67.36-10.01/2+12.01/2 = 21.50mm
Estimated Accuracy 2 Height Gauge 2 Micrometer
0.02+0.02+0.01+0.01 = ±0.06mm
a = 21.50 ± 0.3% mm

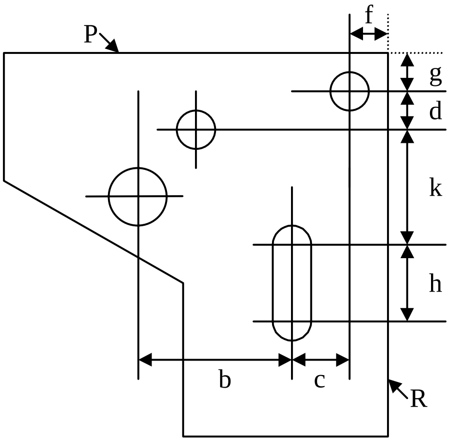
n and m can now be calculated since we have a and ∠an = 30º

m = asin(30º) = 10.75mm ± 0.06mm   m = 10.75 ± 0.6% mm
n = acos(30º) = 18.62mm ± 0.06mm   n = 18.62 ± 0.4% mm



The lengths are simply measured with the height gauge with the workpiece placed on the bench.

R = 146.38 - 46.40 = 99.98mm
P = 146.38 - 46.40 = 99.98mm
Accuracy: 2 height gauge = 0.02 + 0.02 = ±0.04mm
R = 99.98 ± 0.04% mm
P = 99.98 ± 0.04% mm



f and g are measured using the height gauge from sides R and P respectively to the top of the plug gauge (d_3).

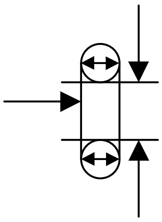
d_3 = 6.01 mm

Giving:
$g + r_3$ = 57.26 - 46.40
g = 7.885 ± 0.05 mm
g = 7.885 ± 0.6% mm

$f + r_3$ = 57.40 - 46.40 f = 7.995 ± 0.05 mm
f = 7.995 ± 0.6% mm

d is determined by measuring from side P to the centre of plug gauge d_1 (used earlier)
Giving:
d = (62.76 – d_1/2) – (57.26 – d3/2) = 3.50 ± 0.06 mm
d = 3.50 ± 1.7% mm

To find dimensions of and relative to the slot, roller gauges are used with d_{roller} = 8.00±0.01mm. This was checked with the micrometer which gave a reading of 8.00mm.



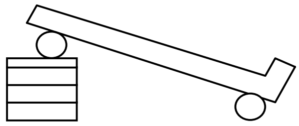
The height is then measured from each side to the top of the roller. And from the side of the slot to side R.
The width of the slot (w_{slot}) is measured with the calipers as 8.00±0.05mm

The heights are measured with the height gauge as 87.90-46.40mm from side P and 87.90–46.40mm from opposite side. The distance from side R to the side of the slot is measured as 74.38-46.4mm.
This gives:
h = R – (87.90 – 46.4) – (87.90 – 46.4) + 8.00 = 24.98±0.09mm
h = 24.98 ± 0.4% mm

k = (87.90 – 46.40) – r_{roller} – g – d = 41.50 – 4.00 – 7.885 – 3.50 = 26.115±0.16mm
k = 26.115 ± 0.6% mm

b and c must now be found:
The distance to the side of the slot is measured as 74.38-46.4mm so:
c = (74.38-46.4) + 8.00/2 - f
c = 74.38-46.40+4.00-7.995 = 23.985±0.14mm = 23.985 ± 0.6% mm
The height to plug gauge d_2 is found to be 122.40 – 46.40.
This gives:
b = (122.40-46.40) – 12.01/2 – (74.38-46.4) – 4.00 = 38.015±0.14mm
b = 38.015 ± 0.4% mm

Angle S must be found using the sine bar:



The bar is built up with slip gauges until the workpiece is level. This is checked using the dial gauge on the height meter. The slip gauges are measured using a micrometer and the required length is found to be 50.00mm the length of the sine bar is known to be 100.00mm so the angle is:

$$\sin^{-1}\left(\frac{50.00}{100.00}\right) = 30.0^{\circ} \text{ which is expected.}$$

Inaccuracies in the sine bar measurement are caused be the inaccuracies in dimensions of the sine bar and the gauge blocks but also by the pressure gauge used to test if the workpiece is level.

In summary (all dims in mm):

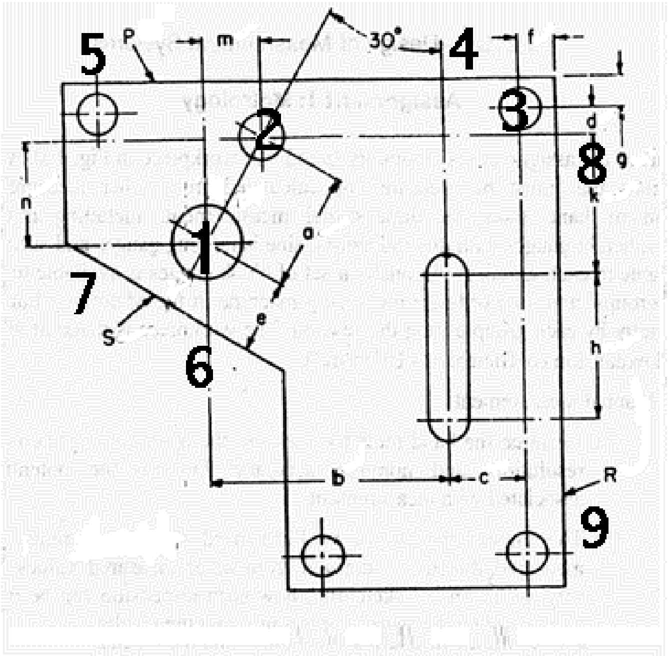
Measurment	a	b	c	d	e	f	g	h	k	n	m	P	R	S
Manual	21.500	38.015	28.985	3.500	14.995	2.995	7.885	24.980	26.115	18.620	10.750	99.980	99.980	30º

It is noticed that the errors associated with the measuring devices add up considerably when dimensions are calculated from many measurements. It is therefore more accurate to measure dimensions directly whenever possible.

2/ CMM Measurement

CMM stands for co-ordinate measuring machine. It is a machine that employs three moveable components that travel along mutually perpendicular guideways to measure a workpiece by determining the X, Y and Z. co-ordinates of points on the workpiece using a contact or non-contact probe and displacement sensors. Modern CMMs can automatically determine dimensions of a given workpiece and upload the information to a computer. The CMM used will however only give the output as XYZ co-ordinates it will be of the cantilever type. CMMs can suffer from relatively large Abbe's offset due to the distance of the measuring point from the line of measurement. This is unavoidable if large displacements are to be permitted.
CMMs require a large capital outlay but greatly simplifies and automate the measuring process, giving a greater accuracy than traditional hand-tools.

The following measurements will need to be taken:



The workpiece is bolted to CMM bench to ensure it does not move while measurement are being taken. The position of the holes is measured using a cone tool. This allows XY co-ordinates to be measured (Z-axis is ignored). The measurements on the edge of the workpiece are made using a cylindrical 10mm probe. This has a accuracy of about ±0.001mm. The slot cannot be measured due to the absence of a suitable tool. One of the holes (First measurement) will be the reference point (0.000,0.000). The measurements taken are shown below Only the X and Y measurements are needed, the Z-axis can be ignored.

Measurment	X (mm)	Y (mm)
1	+0.000	+0.000
2	-9.620	+18.864
3	-60.798	+24.698
4	-58.060	+37.430
5	+14.018	+34.188
6	+0.060	-22.812
7	+27.358	-8.252
8	-73.914	+20.668
9	-77.004	-50.302

Most of the measurements can be found using simple trigonometry (Pythagoras) or co-ordinate geometry.

Find a:

$$\sqrt{(-9.620-0.0)^2+(18.864-0.0)^2}=21.175mm$$

Since we know angle (an) is 30º we can find m and n:
m = asin(30º) = 10.587mm
n = acos(30º) = 18.338mm

To find g d and we need the equation of side (line) P.
y = mx + c y = -0.044979x + 34.8185

We now need the distance of point 3 from this line:
This used simple co-ordinate geometry, by finding the line perpendicular the this line that passes through point 3 it is possible to solve the equations to find a point. The distance from this point to point 3 is the dimension g plus the offset created by the probe (radius 5mm).
Line representing edge y=-0.0449791x + 34.8185
Perpendicular Line y=22.2326x + 1376.39
Point on edge (x,y)=(-60.221,37.5272)
Point 3 (x,y)=(-60.798,24.698)
Distance = 12.8422
Distance - offset = 7.84217
g=7.842mm

Similarly for dimension f:
Line representing edge y=22.9676x + 1718.3
Perpendicular Line y=-0.0435395x + 22.0509
Point on edge (x,y)=(-73.7141,25.2604)
Point 3 (x,y)=(-60.798,24.698)
Distance = 12.9283
Distance - offset = 7.92829mm
f = 7.928mm

To find d the position of point 2 from side P is needed:
Line representing edge y=-0.0449791x + 34.8185
Perpendicular Line y=22.2326x + 232.741
Point on edge (x,y)=(-8.88441,35.2181)
Point 3 (x,y)=(-9.62,18.864)
Distance = 16.3707
Distance - offset = 11.3707
d = 11.3707 – g = 11.3707 – 7.842 = 3.529mm

Finally to find e we need the distance of point 1 from side S:
Line representing edge y=0.533372x + -22.844
Perpendicular Line y=-1.87486x + 0
Point on edge (x,y)=(9.48579,-17.7845)
Point 3 (x,y)=(0,0)
Distance = 20.1561
Distance - offset = 15.1561
Therefore e = 15.156mm

The machine quotes an accuracy of ±0.002mm per co-ordinate. However this was not achieved. The zero position was re-measured after all measurements were taken and was found to be out by 0.1mm in both x and y directions. There are many factors that could lead to this error. The workpiece may have moved slightly due to heating or other factors. The arm of the machine was being moved by hand possibly leading to distortion due to mechanical vibration. There was no way of applying the same pressure each time and the tools in use were worn.

Measurements in summary (all dimensions in mm):

Measurment	a	d	e	f	g	n	m
CMM	21.175	3.529	15.156	7.928	7.842	18.338	10.587

The CMM machine was quick to use but the calculations of the dimensions were slow to perform. This would not be the case with a modern CMM since the output is gathered by manual remote operation or automatically. The data gathered can be processed to give a readout of dimensions, removing the need for manual co-ordinate geometry calculations. A modern, high precision CMM would have a air conditioned atmosphere and the user would operate the machine remotely. This would minimise errors due to thermal drift. The large difference in zero positions is unlikely using these techniques. A CMM machine should have a repeatability of about 0.5μm.

3/ Comparison of CMM and Manual Measurement and Conclusion

The following table shows the hand-tool measurement, the CMM measurement and the difference between them: (dimensions in mm)

Measurment	a	b	c	d	e	f	g	h	k	n	m
Manual	21.500	38.015	23.985	3.500	14.995	7.995	7.885	24.980	26.115	18.620	10.750
CMM	21.175	-	-	3.529	15.156	7.928	7.842	-	-	18.338	10.587
Diference	0.325	-	-	- 0.029	- 0.161	0.067	0.043	-	-	0.282	0.163

The difference is up to 0.3mm for some dimensions. This could be explained by the difference in the zero position on the CMM between the start and end measurements.
This however only applies to measurements a, n and m which were all taken from two co-ordinates. If one of these measurements were incorrect all three dimensions would be affected. Hand tool measurements d, e, f and g are all within or around their error tolerance compared to the CMM measurements.
In theory the CMM should be much more accurate than the hand-tools since a typical CMM would have an accuracy of ±0.002mm and a repeatability of ±0.0005mm.
The height gauge can be considered to be quite accurate since it has the pressure gauge to ensure constant pressure is applied throughout the measurements. When the zero point on this gauge was tested it was found not o have moved. An accuracy of ±0.02mm therefore seems reasonable.
Measurements using the hand tools took longer than using the CMM. However the CMM took some time to set up so if only one or two measurements were required the hand-tools would have been the quicker option. The flexibility o the hand tools is greater than that of the CMM used. The CMM required the workpiece to be bolted down to the table and then to tools (probes) available for measuring were limited resulting in the slot not being measurable. However a fully equipped CMM should be able to measure all feasible dimensions effectively. The hand tools in conjunction with rollers, ball bearings, block gauges and a sine bar should also be able to measure ale feasible dimensions and no problems were encountered with measurement in the laboratory. The relative flexibility of using hand tools is increased when larger workpieces are to be measured. This is especially relevant if the workpiece must be moved a long distance to reach the CMM.
The CMM used was of the cantilever type. Some advantages of this type of CMM are. It is easy to load and unload work pieces. It can measure a work piece that protrudes out of the ends of the measuring table. A disadvantages are if the work piece needs to be manhandled onto the table as using a crane to load the work piece may damage the protruding beam. The machine can be operated from the front and sides but operating

it from behind is difficult. The y-axis beam is also being supported on only one side it flexes more, thus creating a greater Abbe's off-set error than the machines whose beams are supported on both sides. Accuracy maintenance is also difficult.
A CMM machine is also very useful for mass produced items because it can be connected up to a system of computers to aid and automated with Computer Aided Design/Control.
Manual measurement is generally less accurate than CMM. The processes involved can lead to large errors. However since a CMM will cost a business many thousands of pounds it is not always feasible for smaller companies where hand tools would be used. Even in a large company CMM is limited only to high precision measurements. It is quicker to use hand tools initially than have to move to workpiece to the room with the CMM and load and measure it.

From the results and analysis it can be seen that a CMM is the more accurate option and also the quickest if many measurements must be taken. However for some measurements hand tools can still be the more flexible and quicker option.

Part 2 – Performance Evaluation

(a) Quoted accuracy = ± 0.04mm
Quoted Repeatability = ±0.02mm

Accuracy = $\frac{1}{n}\sum_{i=1}^{n} e_i = \bar{x} - x_0$

Accuracy = $\frac{\sum xi}{n} - 100$ = 1485.001/15 – 100 = -0.999mm

Repeatability = 3σ = $3\left(\sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}\right)$ = 3*0.004383 = 0.0131mm

(b)
Accuracy is the degree of agreement of the measured value with its absolute true value.
Repeatability of precision is the ability of an instrument to give identical values, or responses, when the same input is applied repeatedly over a short period of time with:

- Same measurement conditions.
- Same instrument
- Same observer
- Same location
- Same conditions of use.

Simply, accuracy is how close the measured value is to the actual value and repeatability is a measure of the variability of the measured value.

The accuracy of the machine is very poor since the value is over 24 times the stated accuracy. This means the average diameter produced is 99.00mm rather than 100.0mm. The accuracy of the machine is clearly not as stated and if their

investigation is without flaws the machine should be sent for checking/recalibrating. The repeatability of this machine is within the stated range, therefore the variability of the sizes produced will be small and to the manufactures specifications.
Their results show the machine will consistently produce smaller diameter holes but will consistently produce them therefore the machine is not to the manufactures specifications.

(c) The production manager should contact the manufacture and arrange for a recallibration an error of 1mm is not acceptable. In the short term a compensation of 1mm extra should produce diameters closer to 100mm (i.e. enter a diameter of 101mm into the machine).

2.
Compound error:

$$\delta\theta = \frac{\delta\theta}{\delta h}\delta h + \frac{\delta\theta}{\delta l}\delta l$$

$$\frac{\delta\theta}{\delta h} = \frac{1}{l\cos\theta} \qquad \frac{\delta\theta}{\delta l} = -\frac{h}{l^2\cos\theta}$$

$$\delta\theta = \frac{1}{l\cos\theta}\delta h - \frac{h}{l^2\cos\theta}\delta l$$

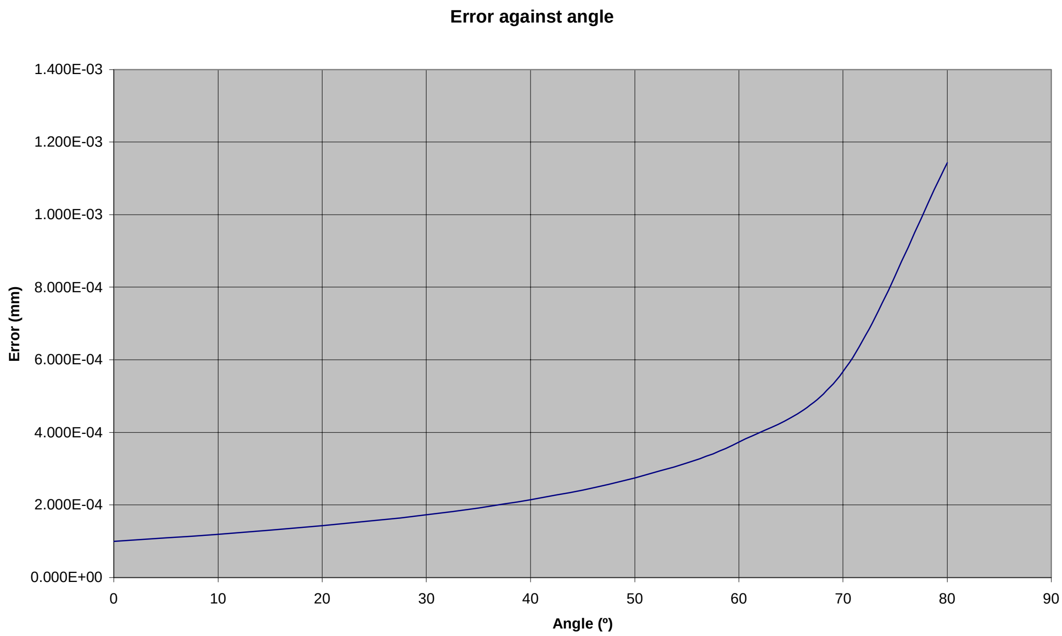
h = lsinθ So:

h = lsinθ So: $\delta\theta = \frac{1}{l\cos\theta}\delta h + \frac{\sin\theta}{l\cos\theta}\delta l$

$$\delta\theta = \frac{1}{l\cos\theta}\delta h + \frac{\tan\theta}{l}\delta l$$

(b) l = 100mm. Estimates used for δl and δh.

Angle (°)	l (mm)	δh (mm)	δl (mm)	Error (mm)
0	100	0.01	0.01	1.000E-04
10	100	0.01	0.01	1.192E-04
20	100	0.01	0.01	1.428E-04
30	100	0.01	0.01	1.732E-04
40	100	0.01	0.01	2.145E-04
50	100	0.01	0.01	2.747E-04
60	100	0.01	0.01	3.732E-04
70	100	0.01	0.01	5.671E-04
80	100	0.01	0.01	1.143E-03



A sine bar is only used for angles less than 45º. As can be seen from the graph the error increases rapidly after 45º.

(c) h = l*sin(θ) h = 200 * sin(25º20’08”)
h = 85.584mm

$$\delta\theta = \frac{0.005}{200 * \cos(25^{o}20'8'')} + \frac{\tan(25^{o}20'8'')}{200} * 0.2$$

$\delta\theta = \pm 5.011 \times 10^{-04}$ mm

(d) Alignment and Abbe’s offset errors can result from using a dial gauge when used in conjunction with a sine bar. These can be minimised by ensuring the work is aligned correctly any by keeping the line of measurement as close to the measured point as possible.

Bibliography

Lecture Notes - ES21Q Design of Measurement Systems.
Assignment Sheet - ES21Q Design of Measurement Systems.

Appendix A: Matlab Program for edge to point distance.

```
%x1,x2,x3,y1,y2,y3 set before
m=(y1-y2)/(x1-x2)
c=y1-x1*m
fprintf(1,'Line representing edge y=%gx + %g\n',m,c);
mper=-1/m;
cper=y3-mper*x3;
fprintf(1,'Perpendicular Line y=%gx + %g\n',mper,cper);
xp=(c-cper)/(mper-m);
yp=m*xp+c;
fprintf(1,'Point on edge (x,y)=(%g,%g)\n',xp,yp);
fprintf(1,'Point 3 (x,y)=(%g,%g)\n',x3,y3);
l=sqrt((y3-yp)^2+(x3-xp)^2);
fprintf(1,'Distance = %g\n',l);
fprintf(1,'Distance - offset = %g\n',l-5);
```